ON THE STABILITY OF NONLINEAR CONTROL SYSTEMS WITH INFINITE NUMBER OF DEGREES OF FREEDOM

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We shall investigate control systems with infinite number of degrees of freedom and one nonlinearity, the latter being an operator of definite class. Sufficient conditions of stability of the noncritial and critical (one zero root) cases are obtained in terms of frequency.

1. While the problem of absolute stability of nonlinear control systems with a finite number of degrees of freedom (systems with concentrated parameters) is discussed in a number of monographs [1 to 3], the analogous problem for the systems with finite number of degrees of freedom (systems with disturbed parameters), is the subject of only a few papers [4 to 7]. In the present work as in [6 and 7] it is assumed that the system contains only one nonlinearity, and can be fully described by

$$\sigma(t) = f(t) - \int_0^t \gamma(t-\tau)\, \eta(\tau)\, d\tau \qquad (1.1)$$

Here $\eta(\tau)$ is a strongly continuous hysteresis function [8], i.e. a nonlinear operator, whose action ensures that a continuous correspondence exists between any $t > 0$, any function $\sigma(\tau)$ continuous on $[0, t]$ and any initial value $\varphi_0 \in E[\sigma(0)]$ (where $E[\sigma(0)]$ is the set of initial values of the hysteresis function), and a function

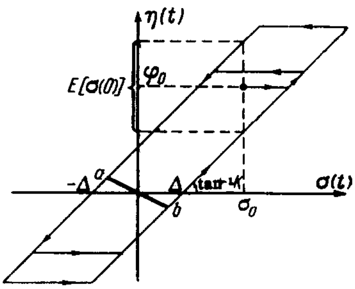

Fig. 1

$$\eta(\tau) = \varphi[\sigma, \varphi_0]_\tau, \qquad \eta(0) = \varphi_0 \qquad (1.2)$$

continuous in $[0, t]$. Ordinary, continuous single-valued function $\varphi(\sigma)$ is a particular example of such a hysteresis function, and in this case $E[\sigma(0)] \equiv \varphi[\sigma(0)]$. In (1.1) function $f(t)$ is independent of η and describes characteristic oscillations of the linear part of the system.

Laplace transform of the kernel

$$\gamma(t), \qquad \chi(p) = \int_0^\infty e^{-pt}\gamma(t)\, dt$$

shall be called a transfer function of the linear part of the system and $\chi(i\omega)$ will be its frequency characteristic.

In [6], absolute stability of the unique position of equilibrium of the system defined by (1.1) was investigated in its critical state of one zero root (function $\chi(p)$ has a pole of first order at zero) and it was assumed that φ is a single-valued continuous function of σ, satisfying at $k=+\infty$, the condition

$$0<\varphi/\sigma<k\leqslant+\infty \tag{1.3}$$

In [7] a noncritical case of the same problem is investigated under the assumption that φ is a strongly continuous hysteresis function, satisfying (1.3).

We ought to mention that hysteresis functions which fail to satisfy (1.3) exist, and a system exhibiting such nonlinearity often possesses a nonunique point of equilibrium. A gap ([1], p.129) (see Fig.1) is a typical example of such a nonlinearity. It emerges in connection with the problem of stability of a set of equilibrium states of a system defined by (1.1) where the nonlinearity (1.2) is a member of a class of strongly continuous hysteresis functions transforming absolutely continuous function $\sigma(\tau)$ whose derivative $\sigma^{\cdot}(\tau), \in L_2(0,t)$, and whose initial value is φ_0, into the absolutely continuous function (1.2) with $\eta^{\cdot}\in L_2[0,t]$, and with the condition

$$0\leqslant\eta^{\cdot}/\sigma^{\cdot}\leqslant k\leqslant+\infty \tag{1.4}$$

fulfilled at almost all instants of time.

Here $^{\cdot}$ denotes differentiation with respect to time, while $\alpha\in L_r(0,t)$ means, that the integral

$$\int_0^t|\alpha(\tau)|^r d\tau$$

converges.

Obviously, a gap with its generator inclined to the abscissa at the angle of $\tan^{-1}K$ (see Fig.1), satisfies the condition (1.4)

2. First we shall investigate the noncritical case, when the functions $f(t)$ and $\gamma(t)$ in (1.1) tend to zero, as $t\to+\infty$. To determine the possible equilibrium states in (1.1), we shall put $\sigma(t)\equiv\sigma_\infty$, $\eta\equiv\varphi_\infty$, make the substitution $-\tau=\lambda$, and pass to the limit as $t\to+\infty$. The above procedure gives

$$\sigma_\infty+\varphi_\infty\int_0^\infty\gamma(\lambda)\,d\lambda=0 \tag{2.1}$$

which should be satisfied by every equiliblium state $(\sigma_\infty,\varphi_\infty)$. The actual position of equilibrium will be the points of intersection of the straight line (2.1) with the boundary of the hysteresis function (segment ab on the figure).

T h e o r e m 2.1 . Let the following conditions be satisfied:

1) the inequality (1.4) is valid

2) $\lim\limits_{t\to+\infty}f(t)=0,\qquad f^{\cdot}(t),\ \gamma(t),\ \int_t^\infty\gamma(\lambda)\,d\lambda\in L_2[0,+\infty),\qquad \gamma(t)\in L_1[0,+\infty)$

3) such $\delta>0$ exists, that for all real ω, the particular condition

$$K^{-1}+\operatorname{Re}\chi(i\omega)>\delta \tag{2.2}$$

is fulfilled. Then

$$\sigma(t)+\eta(t)\int_0^\infty\gamma(\lambda)\,d\lambda\to0\quad\text{for }t\to+\infty$$

L e m m a 2.1 . If $\alpha(t)$ and $\beta(t)$ belong to $L_2[0,+\infty)$, then

$$\mu(t)+\int_0^t\alpha(t-\tau)\beta(\tau)\,d\tau\to0\quad\text{for }t\to+\infty$$

P r o o f o f L e m m a 2.1 . Expressing the integral as a sum of two integrals with the limits of integration $[0,\ T]$ and $[T,\ t]$ and using Schwartz inequality, we obtain

$$|\mu(t)| \leqslant \Big[\int_0^T \alpha^2(t-\tau)\,d\tau \int_0^T \beta^2(\tau)\,d\tau\Big]^{1/2} + \Big[\int_T^t \alpha^2(t-\tau)\,d\tau \int_T^t \beta^2(\tau)\,d\tau\Big]^{1/2}$$

which, after substitution $t - \tau = \lambda$, becomes

$$|\mu(t)| \leqslant \Big[\int_{t-T}^{\infty} \alpha^2(\lambda)\,d\lambda \int_0^{\infty} \beta^2(\tau)\,d\tau\Big]^{1/2} + \Big[\int_0^{\infty} \alpha^2(\lambda)\,d\lambda \int_T^{\infty} \beta^2(\tau)\,d\tau\Big]^{1/2}$$

Keeping T such that the second term is smaller than 0.5ϵ (ϵ is arbitrary), we can choose such $T_1 > T$, for which the first term is smaller than 0.5ϵ when $t > T_1 - T$ and this proves the Lemma.

P r o o f o f T h e o r e m 2.1 . Putting $t - \tau = \lambda$ in (1.1) we obtain, in accordance with (1.2)

$$\sigma(t) = f(t) - \int_0^t \gamma(\lambda)\,\eta(t-\lambda)\,d\lambda$$

Differentiation with respect to t and return to the former variable of integration results in

$$\sigma^{\cdot}(t) = \psi(t) - \int_0^t \gamma(t-\tau)\,\eta^{\cdot}(\tau)\,d\tau \qquad (\psi(t) = f^{\cdot}(t) - \gamma(t)\,\varphi_0) \tag{2.3}$$

Let us now introduce the notation

$$\mu^*(t) = \begin{cases} \eta^{\cdot}(t) & (0 \leqslant t \leqslant T) \\ 0 & (t < 0,\ t > T) \end{cases} \qquad \Big(\sigma_1(t) = \int_0^t \gamma(t-\tau)\,\mu^*(\tau)\,d\tau\Big) \tag{2.4}$$

and consider the functional

$$r = \int_0^{\infty} \Big(\sigma^{\cdot}\mu^* - \frac{\mu^{*2}}{k}\Big)\,dt \tag{2.5}$$

By (2.3), $\sigma^{\cdot} = \psi - \sigma_1$ when $0 < t < T$. Hence

$$r = \int_0^T \psi\mu^*\,dt - \int_0^{\infty} \Big(\sigma_1\mu^* + \frac{\mu^{*2}}{k}\Big)\,dt \tag{2.6}$$

The last integral of (2.6) can, by (2.4), Parseval equality and the contraction theorem [9], be transformed into

$$J = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Big(\mathrm{Re}\,\chi(i\omega) + \frac{1}{k}\Big)\,|\zeta(i\omega)|^2\,d\omega \tag{2.7}$$

where $\zeta(p)$ is a Laplace transform of $\mu^*(t)$. By (2.2) and Parseval equality, we have

$$J \geqslant \delta \int_0^{\infty} \mu^{*2}\,dt \tag{2.8}$$

(1.4) and (2.6) yield the inequality

$$\Big|\int_0^T \psi\mu^*\,dt\Big| \leqslant \frac{2}{\delta}\int_0^{\infty} \psi^2\,dt + \frac{\delta}{2}\int_0^T |\eta^{\cdot}|^2\,dt \tag{2.9}$$

from which, together with (2.7) and (2.8) it follows, that when $r \geq 0$,

$$\int_0^T |\eta^\cdot|^2\, dt \leqslant C \qquad \left(C = \frac{4}{\delta^2}\int_0^\infty \psi^2\, dt\right) \tag{2.10}$$

In the above estimate, constant C is independent of T. Consequently

$$\eta^\cdot \in L_2\,[0, +\infty)$$

Returning to (1.1) we obtain, after integration by parts,

$$\sigma(t) = f(t) - \int_0^t \eta(\tau)\frac{d}{d\tau}\int_{t-\tau}^\infty \gamma(\lambda)\, d\lambda = f(t) + \varphi_0\int_t^\infty \gamma(\lambda)\, d\lambda -$$
$$- \eta(t)\int_0^\infty \gamma(\lambda)\, d\lambda + \int_0^t d\tau\, \eta^\cdot(\tau)\int_{t-\tau}^\infty \gamma(\lambda)\, d\lambda$$

In the right-hand side of the above expression, first, second and fourth terms tend to zero as $t \to \infty$ by the condition 2 of the Theorem 2.1 and Lemma 2.1, and this completes the proof.

We should note, that for systems with a finite number of degrees of freedom, the above Theorem follows from the results obtained in [10].

Now, in order to obtain (2.2) in a more general form, let us consider the following conditions.

M) There exist a constant m_φ such, that for any $\sigma(t)$ belonging to the region of definition of the nonlinear operator $\eta(t) = \varphi[\sigma, \varphi_0]_t$ and any $t > 0$,

$$|\eta(t) - k\sigma(t)| \leqslant m_\varphi \tag{2.11}$$

N) If $|\sigma^\cdot(t)|$ is uniformly bounded in $t > 0$, then the functional

$$\int_0^t \sigma^{\cdot\cdot}(\tau)\, \eta^\cdot(\tau)\, d\tau$$

is also uniformly bounded from above in $t > 0$.

L) If the nonlinear equation under consideration is linearized by substitution $\eta = k\sigma + \nu(t)$, where $\sup|\nu(t)| < \infty$, for $t > 0$, then the solution $\sigma(t)$ of the resulting linear equation will be uniformly bounded for $t > 0$.

Condition (2.11) is obviously fulfilled for the gap with the generator inclined at the angle of $\tan^{-1} k$. It is easy to show, that for such a gap,

$$\int_0^t \varphi^\cdot \sigma^{\cdot\cdot}\, d\tau \leqslant \frac{k}{2}\sigma^{\cdot 2}(t)$$

therefore condition (N) is also fulfilled.

For the systems with a finite number of degrees of freedom fulfilment of (N) follows from (2.2) (and from a more general condition (2.12) given below), by the Nyquist criterion and obvious estimates.

T h e o r e m 2.2. We shall assume that:

1) Conditions (1.4) together with (M), (N) and (L), are fulfilled;

2) $\lim\limits_{t\to+\infty} f(t) = 0,\ f^\cdot, f^{\cdot\cdot}, \gamma, \gamma^\cdot, \int_t^\infty \gamma\, d\lambda \in L_2\,[0, +\infty),\ \gamma \in L_1\,[0, \infty),\ \sup\limits_{t>0} |f^\cdot| < +\infty$

3) There exist such $\delta > 0$ and $\vartheta \geqslant 0$, that for all real ω the frequency condition

$$\frac{1}{k} + \operatorname{Re}\underset{\infty}{[}(1 - i\omega\vartheta)\chi(i\omega)] > \delta \tag{2.12}$$

is true.

Then,

$$\sigma(t) + \eta(t)\int_0^\infty \gamma(\lambda)\, d\lambda \to 0 \quad \text{for } t \to +\infty$$

P r o o f . Having introduced the functions (2.4), we shall now consider the functional

$$r = \int_0^\infty \left(\sigma^{\cdot}\mu^* - \frac{\mu^{*2}}{k} - \vartheta\sigma^{\cdot\cdot}\mu^*\right) dt \tag{2.13}$$

Using (1.1), Parseval equality and the conditions (1.4) and (2.12), and after some calculations similar to those performed in the proof of Theorem 2.1, we obtain the estimate

$$\frac{\delta}{2}\int_0^T \eta^{\cdot 2}\, dt \leqslant \frac{2}{\delta}\int_0^\infty (\psi - \vartheta\psi^{\cdot})^2\, dt + \vartheta\int_0^T \sigma^{\cdot\cdot}\eta^{\cdot}\, dt \tag{2.14}$$

Substituting $\eta = k\sigma + \nu$ in (1.1) and utilizing the conditions (M) and (L) we conclude that $|\sigma(t)|$ and consequently by (2.11) also $|\eta(t)|$, are uniformly bounded for $t > 0$. Differentiating (1.1) we find, that $|\sigma^{\cdot}(t)|$ is also uniformly bounded for $t > 0$. Hence, it follows from (N), that the second integral in the right-hand side of (2.14) is uniformly bounded from above in T. Consequently $\eta^{\cdot} \in L_2[0, +\infty)$. Remainder of the proof follows that of Theorem 2.1.

We should note that, if the functions $f(t)$ and $\gamma(t)$ as well as their derivatives obtained under the conditions (2) of the Theorems 2.1 and 2.2 decay exponentially as $t \to +\infty$ (this occurs in case of systems with finite number of degrees of freedom, whenever the roots of characteristic equation lie on the open left-hand side semi-plane), then the conditions (2) of the Theorems 2.1 and 2.2 are obviously fulfilled.

3. Now we shall consider the critical case of one zero root. Our equation is

$$\sigma(t) = f(t) + \varkappa - \int_0^t [\gamma(t-\tau) + \rho]\,\eta(\tau)\, d\tau \tag{3.1}$$

where $\varkappa$ and ρ are constants, $f(t)$ and $\gamma(t)$ are decaying functions and η is a nonlinear operator described at the beginning of this paper. Having introduced the variable

$$\xi(t) = \frac{\varkappa}{\rho} + \int_0^t \eta(\tau)\, d\tau$$

we shall write (3.1) as

$$\sigma(t) = f(t) - \int_0^t \gamma(t-\tau)\,\eta(\tau)\, d\tau - \rho\xi(t), \quad \xi^{\cdot} = \eta \tag{3.2}$$

In order to determine the possible positions of equilibrium of the system (3.2) we shall put $f \equiv 0$, $\sigma(t) = \sigma_\infty$, $\eta(t) \equiv \varphi_\infty$, and $\xi(t) \equiv \xi_\infty$. From the second equation of (3.2) we obtain $\varphi_\infty = 0$, while the first one yields the equality $\sigma_\infty + \rho\xi_\infty = 0$.

We shall also assume here that two continuous functions $\varphi_+(\sigma)$ and $\varphi_-(\sigma)$ together with the numbers σ' and σ'' exist such, that for all $t > 0$

$$\varphi_-[\sigma(t)] \leqslant \eta(t) \leqslant \varphi_+[\sigma(t)], \quad \varphi_-(\sigma) > 0 \ \text{for } \sigma > \sigma'', \quad \varphi_+(\sigma) < 0 \ \text{for } \sigma < \sigma' \tag{3.3}$$

For the gap (see Fig.1) we can obviously assume, that

$$\varphi_+(\sigma) = k(\sigma + \Delta), \ \text{a} \ \varphi_-(\sigma) = k(\sigma - \Delta)$$

T h e o r e m 3.1 . We shall assume that

1) conditions (1.4) and (3.3) hold;

2) $\lim\limits_{t\to+\infty} f = \lim\limits_{t\to+\infty} f^{\cdot} = \lim\limits_{t\to+\infty} \gamma = 0, \quad f^{\cdot}, \gamma \in L_2[0, +\infty), \quad \gamma \in L_1[0, +\infty);$

3) $\rho > 0$ and such $\delta > 0$ exists, that for all real ω condition (2.2) holds. Then

$$\lim \eta = \lim (\sigma + \rho\xi) = 0 \quad \text{for } t \to \infty$$

L e m m a 3.1.. If $\alpha(t) \in L_1[0, +\infty)$, while $\beta(t)$ is uniformly bounded in $t > 0$ and tends to zero as $t \to \infty$, then

$$\mu(t) = \int_0^t \alpha(t-\tau)\beta(\tau)\,d\tau \to 0 \quad t \to +\infty.$$

Proof of this Lemma follows closely that of of the previous Lemma.

P r o o f o f T h e o r e m 3.1 . Using the functions (2.4) and the functional (2.5), we shall transform the latter, utilizing the equality $\sigma^\cdot = \psi - \sigma_1 - \rho\xi^\cdot$ valid for $0 < t < T$ (see the proof of Theorem 2.1)

$$r = \int_0^T \psi\mu^* \,dt - \rho \int_0^T \xi^\cdot \eta^\cdot \,dt - \int_0^\infty \left(\sigma_1\mu^* + \frac{\mu^{*2}}{k}\right) dt \tag{3.4}$$

For the first integral of the right-hand side of (3.4), estimate (2.9) holds. Second term, by virtue of the second equation of (3.2), is equal to $0.5\rho(\varphi_0^2 - \eta^2(T))$. Last term, by Parseval equality and condition (2.2), has an upper bound

$$-\delta \int_0^T \eta^{\cdot 2}\,dt$$

Summing the above results and using (3.4) with (1.4), we obtain

$$\frac{\rho}{2}\eta^2(T) + \frac{\delta}{2}\int_0^T \eta^{\cdot 2}\,dt \leqslant \frac{\rho}{2}\varphi_0^2 + \frac{2}{\delta}\int_0^\infty \psi^2\,dt$$

from which it follows that $|\eta(t)|$ is uniformly bounded in $t > 0$, and $\eta^\cdot \in L_2[0, +\infty)$.

Differentiating the first equation of (3.2) and utilizing its second equation, condition (2) of Theorem 3.1 and Lemma 2.1, we obtain

$$\lim_{t \to +\infty} (\sigma^\cdot + \rho\eta) = 0 \tag{3.5}$$

Next we shall show that $\lim \eta = 0$ as $t \to +\infty$. By (3.5), for any $\delta > 0$ we can find such $T > 0$, that for all $t > T$, the estimate $|\varepsilon|(t)| < 0.5\delta$ is true for the function $\varepsilon(t) = \sigma^\cdot + \rho\eta$. Let $\rho\eta(T) > \delta$ (trend of the argument is analogous for the case $\rho\eta(T) < -\delta$) . Then $\sigma^\cdot = \varepsilon - \rho\eta < -0.5\delta$ for $t > T$ as long as $\rho\eta > \delta$. By (3.3) we can find such an instant of time $t_1 > T$, that when $t = t_1$

$$-\delta < \rho\eta(t) < \delta \tag{3.6}$$

It remains to show that if the estimate (3.6) is true for $t = t_1 > T$, then it will hold for all $t > t_1$. Let us assume that $|\rho\eta(t_2)| > \delta$ at $t_2 > t_1$, e.g. $\rho\eta(t_2) > \delta$. Hence, such an instant of time $t_0 \in (t_1, t_2)$, can be found for which $\rho\eta(t) > \delta$ when $t = t_0$, and at which the derivatives $\sigma^\cdot$ and $\eta^\cdot(\eta^\cdot > 0)$ exist. Then, by (1.4), $\sigma^\cdot(t_0) > 0$ and $\varepsilon(t_0) > \delta$ which contradicts the choice of T . Hence, $\lim \eta = 0$ as $t \to +\infty$.

Passing to the limit as $t \to +\infty$ in the first equation of (3.2) and using Lemma 3.1, concludes the proof of Theorem 3.1.

A result resembling the above theorem was obtained in [11 to 13] for systems with finite number of degrees of freedom.

Condition (2.2) can be extended to the critical case, namely the following theorem is true.

T h e o r e m 3.2 . Let us assume that for the equation under consideration

1) conditions (1.4),(3.3),(M),(N) and (L) are satisfied.

2) $\lim_{t \to +\infty} f = \lim_{t \to +\infty} f^\cdot = 0, \quad f^\cdot, f^{\cdot\cdot}, \gamma, \gamma^\cdot \in L_2[0, +\infty), \quad \gamma \in L_1[0, +\infty)$

3) $\rho > 0$ and such $\delta > 0$ and $\vartheta \geqslant 0$, exist, that for all real ω

$$1 / K - \vartheta\rho + \operatorname{Re}\,[(1 - i\omega\vartheta)\,\chi\,(i\omega)] > \delta \tag{3.7}$$

Then

$$\lim \eta = \lim\,(\sigma + \rho\xi) = 0 \qquad \text{for} \quad t \to + \infty$$

P r o o f . Using (2.4) and (3.2) we shall transform the functional (2.13). Steps similar to those employed in the proof of Theorem 3.1, lead to the estimate

$$\frac{\rho}{2}\,\eta^2(T) + \frac{\delta}{2}\int_0^T \eta^{\cdot 2}\,dt \leqslant \frac{\rho}{2}\,\varphi_0^2 + \frac{2}{\delta}\int_0^\infty (\psi - \vartheta\psi^{\cdot})^2\,dt + \vartheta\int_0^T \sigma^{\cdot\cdot}\eta^{\cdot}\,dt \tag{3.8}$$

Putting in (3.1) $\eta = k\delta + \nu$ we find that, by (M) and (L), $|\sigma(t)|$ and consequently $|\eta(t)|$ are uniformly bounded when $t > 0$. Differentiation of the first equation of (3.2) shows that $|\sigma^{\cdot}(t)|$ is also uniformly bounded. Hence, from (3.8), by virtue of (L) it follows that $\eta^{\cdot} \in L_2[0, +\infty)$. Rest of the proof is identical to that of Theorem 3.1 and this completes the proof of Theorem 3.2.

We should note that in case of systems with finite number of degrees of freedom, frequency conditions quoted in Theorems 2.1, 2.2, 3.1 and 3.2 can [3] be expressed in terms of Lur'e's resolvents. These frequency conditions possess a simple geometrical meaning: modified phase-amplitude frequency characteristic of the linear part of the system should be contained in some semi-plane.

Examination of the validity of conditions (2.2),(2.12) and (3.7) by analytical methods is, as a rule, very difficult, and can be performed only for relatively simple cases. Otherwise, numerical methods must be used.

All the above results can be extended to systems with more than one nonlinearity but corresponding frequency conditions become difficult to verify.

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SCHWARZSCHILD PROBLEM FOR A METRIC WHOSE SPATIAL PART IS PURELY EUCLIDEAN

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In the course of investigation of motion of a material medium, use is often made of curvilinear coordinates. In such a case, the metric of curved space is given by the interval

$$-ds^2 = g_{ik}\, dx^i\, dx^k$$

where g_{ik} are the components of the metric tensor.

In the case of motions possessing central symmetry (after some coordinate transformation, should it be necessary), the usual interval is given in the form [1] of a Schwarzschild metric

$$-ds^2 = -e^{\nu} c^2 dt^2 + e^{\lambda} dr^2 + r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2 \qquad (\nu = \nu(t, r),\ \lambda = \lambda(t, r))$$

However, in a number of gas-dynamics problems, a different metric is found to be of use, in which the spatial part of the interval is Euclidean

$$-ds^2 = -e^{\nu} c^2\, dt^2 + 2e^{1/2\mu}\, c\,dt\,dr + dr^2 + r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (\mu = \mu(t, r))$$

With this condition adopted, the investigation of the motion of gas in the gravity field becomes considerably simplified. Purely spatial metric will still be a curve

$$dl^2 = (1 + e^{\mu-\nu})\, dr^2 + r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

and so will geodesic trajectories.

The above metric can be obtained directly from the Schwarzschild's metric by putting $e^{\nu} = 1 - r_0 / r$, $\lambda = -\nu$, and using the transformation